Degree-1 Physics (Hons) Lecture-4 on the topic Damped Forced Oscillations.

Degree-1, Physics (Hons)

Lecture- 4

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Forced Oscillations:-

Oscillations produced by an external periodic driving force are called forced oscillations. There is an increase in the amplitude of oscillations when the driving force is close to natural frequency of the system.

The damped oscillator with harmonic driving force, has the equation of motion

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + Kx = F_0 \cos \omega_D t$$

where $K \rightarrow$ Force constant, $K = m\omega_0^2$, $\omega_0 \rightarrow$ natural frequency

$b \rightarrow$ damping constant

$\omega_D \rightarrow$ driving frequency.

driving force at $t=0$.

driving force at time $= t$

$$F(t) = F_0 \cos \omega_D t.$$

$$\text{Real part of } F(t) = F_0 e^{+i\omega_D t}$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + Kx = F_0 e^{+i\omega_D t} \quad \text{--- (1)}$$

Let's assume the solution of above equation is

$$x(t) = B e^{+i\omega_D t}$$

Now

$$-m\omega_D^2 B e^{i\omega_D t} + b i\omega_D B e^{i\omega_D t} + K B e^{i\omega_D t} = F_0 e^{i\omega_D t}$$

$$B(-m\omega_D^2 + i b\omega_D + K)\cancel{e^{i\omega_D t}} = F_0 \cancel{e^{i\omega_D t}}$$

$$B = \frac{F_0}{(-m\omega_D^2 + i b\omega_D + K)}$$

$$B = \frac{F_0/m}{\left(-\omega_D^2 + \frac{i b\omega_D}{m} + \frac{K}{m}\right)}$$

$$B = \frac{F_0/m}{\left[\left(\frac{k}{m} - \omega_D^2\right) + i\frac{b\omega_D}{m}\right]}$$

$$B = \frac{F_0/m}{\left[\left(\frac{k}{m} - \omega_D^2\right) + i\frac{b\omega_D}{m}\right]} \times \frac{\left[\left(\frac{k}{m} - \omega_D^2\right) - i\frac{b\omega_D}{m}\right]}{\left[\left(\frac{k}{m} - \omega_D^2\right) - i\frac{b\omega_D}{m}\right]}$$

$$B = \frac{F_0/m\left[\left(\omega_0^2 - \omega_D^2\right) - i\frac{b\omega_D}{m}\right]}{\left[\left(\omega_0^2 - \omega_D^2\right)^2 + \frac{b^2\omega_D^2}{m^2}\right]} \qquad \left(\because \omega_0^2 = \frac{k}{m}\right.$$

$$x + iy = r\,e^{i\phi}$$

where $r = \sqrt{x^2+y^2}$, $\tan\phi = y/x$

write

$$\left(\omega_0^2 - \omega_D^2\right) - i\frac{b\omega_D}{m} = \sqrt{\left(\omega_0^2 - \omega_D^2\right)^2 + \frac{b^2\omega_D^2}{m^2}}\; e^{i\phi}$$

where, $\tan\phi = \dfrac{b\omega_D/m}{(\omega^2 - \omega_0^2)}$

Now

$$B = \frac{F_0/m\,\sqrt{\left(\omega_0^2 - \omega_D^2\right)^2 + \frac{b^2\omega_D^2}{m^2}}\; e^{i\phi}}{\left(\omega_0^2 - \omega_D^2\right)^2 + \frac{b^2\omega_D^2}{m^2}}$$

$$B = \frac{F_0/m\,\cancel{\sqrt{\left(\omega_0^2 - \omega_D^2\right)^2 + b^2\omega_D^2/m^2}}\; e^{i\phi}}{\sqrt{\left(\omega_0^2 - \omega_D^2\right)^2 + \frac{b^2\omega_D^2}{m^2}} \times \cancel{\sqrt{\left(\omega_0^2 - \omega_D^2\right)^2 + \frac{b^2\omega_D^2}{m^2}}}}$$

$$B = \frac{F_0/m}{\sqrt{\left(\omega_0^2 - \omega_D^2\right)^2 + \frac{b^2\omega_D^2}{m^2}}}\; e^{i\phi}$$

Now

$$x(t) = B\,e^{i\omega_D t}$$

$$x(t) = \frac{F_0/m}{\sqrt{\left(\omega_0^2 - \omega_D^2\right)^2 + \frac{b^2\omega_D^2}{m^2}}}\; e^{i\phi}\, e^{i\omega_D t}$$

$$x(t) = \frac{F_0/m}{\sqrt{(\omega_0^2-\omega_D^2)^2+\frac{b^2\omega_D^2}{m^2}}}\, e^{i(\omega_D t+\phi)}$$

Taking real part of this

$$\boxed{x(t) = \frac{F_0/m}{\sqrt{(\omega_0^2-\omega_D^2)^2+\frac{b^2\omega_D^2}{m^2}}}\cos(\omega_D t+\phi)}$$

This is the solution of damped forced oscillations.