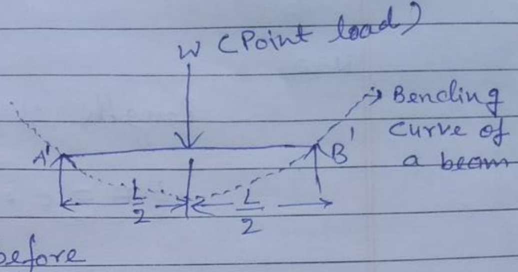
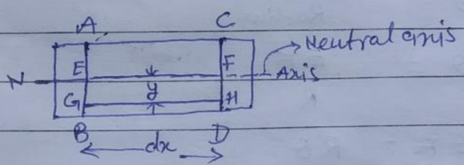
Degree-1 Physics (Hons) Lecture 2 on the topic bending of Beam.

Bending of Beam :->
or Flexural Formula :->

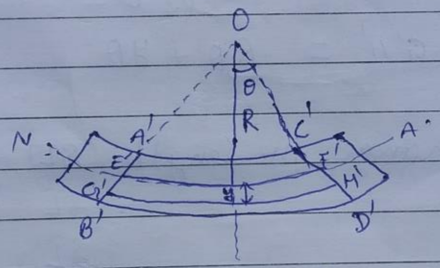


Consider the beam section before bending as shown below



Consider another layer GH at a distance 'y' from neutral axis.

Consider the section of the beam after bending:-



Let the beam may bend as shown in above figure.

R is the radius of curvature of layer E'F'

y is the distance from neutral axis upto layer G'H'

θ is the angle subtended by arc at the centre.

Strain in layer GH due to bending

$$e = \frac{\text{change in length}}{\text{original length}} \Rightarrow \frac{G'H' - GH}{GH}$$

$$\therefore \quad e = \frac{G'H' - EF}{EF} \quad (\because GH = EF) \leftarrow (1)$$

Now,

$\because$ length of arc $E'F'$ or EF is given by

$$EF = E'F'$$ (Neutral axis length remain same before and after bending)

$$\theta = \frac{arc}{Radius}$$

$$\theta = \frac{EF}{R}$$

$$EF = R\theta$$

Similarly,
length of arc $G'H'$ is given by

$$G'H' = (R+y)\theta$$

$$\therefore G'H' = R\theta + y\theta$$

$\therefore$ change in length $(\delta L) = G'H' - EF$

$$\delta L = \cancel{R\theta} + y\theta - \cancel{R\theta}$$

$$\delta L = y\theta$$

put all values in equation (1)

$$e = \frac{y\theta}{R\theta}$$

$$e = \frac{y\cancel{\theta}}{R\cancel{\theta}}$$

$$\boxed{e = \frac{y}{R}}$$

From Hook's law

stress$(\sigma_b) \propto$ strain(e)

$$\sigma_b = Ye$$ ($Y \rightarrow$ Young's modulus)

$$e = \frac{\sigma_b}{Y}$$

$$e = \frac{\sigma_b}{Y}$$

$$\therefore \frac{\sigma_b}{Y} = \frac{y}{R}$$

$$\boxed{\frac{\sigma_b}{y} = \frac{Y}{R}} \quad \text{--- (2)}$$

(Here Y Young's modulus
$y \rightarrow$ distance)

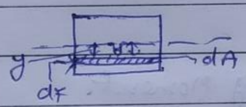
From equation (2) bending stress acting on the layer is given by,

$$\text{Bending stress } \boxed{\sigma_b = \frac{Y}{R} y} \quad \text{--- (3)}$$

From equation (3) we see that Young's modulus (Y) and radius of curvature (R) are constant.

$$\therefore \sigma_b \propto y$$

Consider the cross-section of the beam as rectangular and in that consider an elemental strip having area dA located at a distance 'y' from neutral axis.



Force acting on the elemental strip because of bending

$$dF = \sigma_b \times dA \qquad \left(\because \sigma = \frac{F}{A}\right)$$

$$\therefore dF = \frac{Y}{R} y \, dA$$

Taking moment of force about Neutral axis

(Bending moment) $dM = dF \times y$

$$dM = \frac{Y}{R} y \, dA \times y$$

$$dM = \frac{Y}{R} y^2 \, dA$$

$\therefore$ total moment of the beam about neutral axis

$$\int dM = \int \frac{Y}{R} y^2 \, dA$$

$$M = \frac{Y}{R} \int y^2 \, dA$$

$$M = \frac{Y}{R} \cdot I$$

$\left(\because I = \int y^2 \, dA \right.$
$\left. I \to \text{Moment of inertia} \right)$

$$\therefore \boxed{\frac{M}{I} = \frac{Y}{R}} \quad \text{--- (4)}$$

From Equation (2) and (4)

$$\boxed{\frac{\sigma_b}{y} = \frac{Y}{R} = \frac{M}{I}}$$

or

$$\boxed{\frac{M}{I} = \frac{\sigma_b}{y} = \frac{Y}{R}}$$

This is Flexural Formula
or
Bending Formula

where,

M = Bending moment.
I = Moment of inertia.
σ_b = Bending stress.
y = Distance of layer in which bending is considered from neutral axis,
Y = Young's modulus.
R = Radius of curvature.