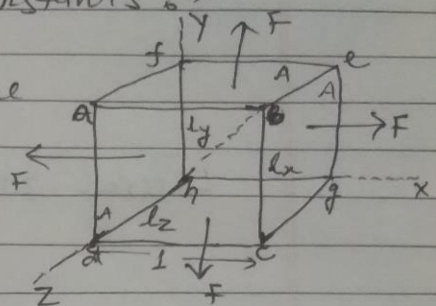
Degree-1 Physics (Hons) Lecture 1 on the topic Relation among Elastic Constants.

D-1 Physics (Hons)

Relation among Elastic Constants :→
(Y, K, η, σ)

Consider a cube of unit volume
i.e $l_x = 1$, $l_y = 1$, $l_z = 1$
volume $V = 1 \times 1 \times 1 = 1$ unit
The area(A) of each face is 1 unit.



we also consider the applied force (F) is 1 unit.

$Y \rightarrow$ Young's modulus, $K \rightarrow$ Bulk modulus
$\eta \rightarrow$ Modulus of rigidity, $\sigma \rightarrow$ Poisson ratio

(1) Relation among (Y, K and σ):

Bulk modulus is define as

$$K = \frac{\text{Volume stress}}{\text{Volume strain}}$$

$$\text{volume stress} = \frac{F}{A}$$

but $F = 1$ unit, $A = 1$ unit.

$$\text{volume stress} = \frac{1}{1} = 1$$

$$\text{volume strain} = \frac{\Delta V}{V}$$

$V = 1$ unit.

$\Delta V = ?$

$$\text{volume strain} = \frac{\Delta V}{1} = \Delta V$$

Now $$K = \frac{1}{\Delta V} \quad \text{——— (1)}$$

longitudinal strain (α) is define as

$$\alpha = \frac{\Delta l}{l}$$

$$l = 1 \text{ unit}$$

$$\alpha = \frac{\Delta l}{1} = \Delta l$$

Lateral strain (β) is define as

$$\beta = \frac{\Delta l}{l} = \frac{\Delta l}{1} = \Delta l$$

If we apply the force along x direction then the new length of the cube along-x will be

$$l_x' = 1 + \alpha - \beta - \beta$$

$$l_x' = 1 + \alpha - 2\beta$$

Similarly if we apply force along Y and z-direction then

$$l_y' = 1 + \alpha - 2\beta$$

$$l_z' = 1 + \alpha - 2\beta$$

Now the change in volume will be

$$\Delta V = V' - V$$

$$V' = l_x' \times l_y' \times l_z' \quad , \quad V = 1 \text{ unit}$$

$$V' = (1 + (\alpha - 2\beta))^3$$

using Bionomial theorem

$$\Delta V' = 1 + 3(\alpha - 2\beta) \qquad \left| (1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \right.$$

$$V' = 1 + 3(\alpha - 2\beta)$$

$$\Delta V = \cancel{1} + 3(\alpha - 2\beta) - \cancel{1}$$

$$\Delta V = 3(\alpha - 2\beta)$$

$$\Delta V = 3\alpha\left(1 - 2\frac{\beta}{\alpha}\right)$$

Poisson ratio $\sigma = \frac{\beta}{\alpha}$

$$\Delta V = 3\alpha(1-2\sigma) \qquad (\because \sigma = \beta/\alpha)$$

$$K = \frac{1}{\Delta V}$$

$$K = \frac{1}{3\alpha(1-2\sigma)}$$

$$K = \frac{1/\alpha}{3(1-2\sigma)} \quad \text{—— (ii)}$$

Young's modulus Y is define as .

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

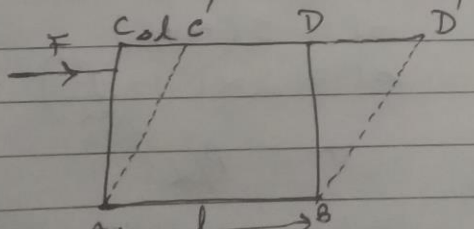
$$Y = \frac{1}{\frac{\Delta l}{l}}$$

$$Y = \frac{1}{\alpha}$$

$$\boxed{K = \frac{Y}{3(1-2\sigma)}} \quad \text{—— (iii)}$$

this is the relation among K, Y and σ.

(ii) Relation among Y, η and σ :⇒



$l = 1$ unit
$A = 1$ unit
$V = 1$ unit
$F = 1$ unit

Modulus of rigidity (η) is define as

$$\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}}$$

$$\eta = \frac{F/l^2}{\frac{\Delta l}{l}}$$