Degree-1 Physics (Hons) Lecture 1.1 on the topic Relation among Elastic Constants.

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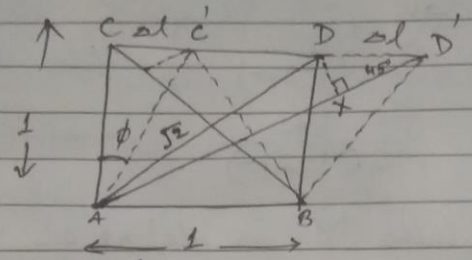
$$\eta = \frac{1}{\Delta l} \quad \text{(*)} \quad (\because F = 1 \text{ unit}, \; l = 1 \text{ unit})$$



Longitudinal strain $(\alpha) = \frac{\Delta l}{l} = \Delta l$

Lateral strain $= \frac{\Delta D}{D}$

change in length $XD' = AD \times (\alpha + \beta)$ $(\because AD = AX)$

$XD' = \sqrt{2}(\alpha + \beta)$

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In $\Delta DD'X$

$$\cos 45^\circ = \frac{XD'}{\Delta l}$$

$$\frac{1}{\sqrt{2}} = \frac{XD'}{\Delta l}$$

$$\Delta l = \sqrt{2} \, XD'$$

$$\Delta l = \sqrt{2} \times \sqrt{2}(\alpha + \beta)$$

$$\Delta l = 2(\alpha + \beta)$$

$$\Delta l = 2\alpha\left(1 + \frac{\beta}{\alpha}\right)$$

$$\sigma = \beta/\alpha.$$

$$\Delta l = 2\alpha(1 + \sigma)$$

$$\frac{1}{\Delta l} = \frac{1}{2\alpha(1+\sigma)}$$

$$\frac{1}{\Delta l} = \frac{1/\alpha}{2(1+\sigma)}$$

$$\frac{1}{\Delta l} = \eta$$

$$\frac{1}{\alpha} = Y$$

$$\frac{1}{\Delta l} = \frac{1/\alpha}{2(1+\sigma)}$$

$$\boxed{\eta = \frac{Y}{2(1+\sigma)}} \quad \text{——— (IV)}$$

(This is the relation among η, Y and σ.

(iii) Relation among Y, K and η :-

we already drived

$$K = \frac{Y}{3(1-2\sigma)} \quad \text{——— (III)}$$

$$\eta = \frac{Y}{2(1+\sigma)} \quad \text{——— (IV)}$$

$$K = \frac{Y}{3(1-2\sigma)}$$

$$1-2\sigma = \frac{Y}{3K} \quad \text{——— (V)}$$

$$\eta = \frac{Y}{2(1+\sigma)}$$

$$2+2\sigma = \frac{Y}{\eta} \quad \text{——— (VI)}$$

adding equation (V) and (VI)

$$3 = \frac{Y}{3K} + \frac{Y}{\eta}$$

$$3 = Y\left(\frac{1}{3K} + \frac{1}{\eta}\right)$$

$$\boxed{\frac{3}{Y} = \frac{1}{3K} + \frac{1}{\eta}}$$ Relation among Y, K and η

(iv) Relation among K, η and σ :-

$$K = \frac{Y}{3(1-2\sigma)} \quad \text{---(iii)}$$

$$\eta = \frac{Y}{2(1+\sigma)} \quad \text{---(iv)}$$

$$K = \frac{Y}{3(1-2\sigma)}$$

$$Y = 3K(1-2\sigma)$$

Put the value of Y in eq^n (iv)

$$\eta = \frac{3K(1-2\sigma)}{2(1+\sigma)}$$

$$2\eta(1+\sigma) = 3K(1-2\sigma)$$

$$2\eta + 2\eta\sigma = 3K - 6K\sigma$$

$$2\eta\sigma + 6K\sigma = 3K - 2\eta$$

$$2\sigma(\eta + 3K) = 3K - 2\eta$$

$$\boxed{\sigma = \frac{3K-2\eta}{2(\eta+3K)}}$$

This is the relation among K, η and σ

or

$$\boxed{K = \frac{(2+2\sigma)\eta}{3(1-2\sigma)}}$$