Degree-2 Physics (Hons) Lecture-5 on the topic Maxwell's Equations & Displacement Current.

Degree-2, Physics (Hons), Paper-III[rd]

Lecture-5

Maxwell's Equations :-

$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$ — (I) — Gauss' Law in electrostatics

$\vec{\nabla}\cdot\vec{B} = 0$ — (II) — Gauss' Law for magnetism

$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ — (III) — Faraday's Law

$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$ — (IV) — Ampere's Law

where,

$\vec{\nabla} \rightarrow$ Del Operator

$\vec{E} \rightarrow$ Electric field intensity.

$\rho \rightarrow$ charge density

$\epsilon_0 \rightarrow$ Permittivity in free space

$\vec{B} \rightarrow$ magnetic field

$\vec{J} \rightarrow$ current density

$\mu_0 \rightarrow$ Permeability in free space.

Integral form of Maxwell's Equations :-

$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$

Taking volume integral on both side

$\int \vec{\nabla}\cdot\vec{E}\, dV = \oint \frac{\rho}{\epsilon_0}\, dV$.

using Gauss' Divergence theorem

$\int \vec{\nabla}\cdot\vec{E}\, dV = \oint \vec{E}\cdot d\vec{S}$

$\boxed{\oint \vec{E}\cdot d\vec{S} = \frac{q}{\epsilon_0}}$ $\quad (\because q = \oint \rho\, dV)$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\int \vec{\nabla}\cdot\vec{B}\, dv = 0$$

$$\boxed{\oint \vec{B}\cdot d\vec{S} = 0}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}.$$

Taking surface integral on both side.

$$\int (\vec{\nabla}\times\vec{E})\, d\vec{S} = \int -\frac{\partial \vec{B}}{\partial t}\, dS.$$

using stoke's theorem.

$$\int (\vec{\nabla}\times\vec{E})\, d\vec{S} = \int \vec{E}\cdot d\vec{l}$$

$$\int \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{S}$$

$$\int \vec{E}\cdot d\vec{l} = -\frac{\partial \phi_B}{\partial t} \qquad \left(\phi_B = \int \vec{B}\cdot d\vec{S}\right)$$

$$\boxed{\int \vec{E}\cdot d\vec{l} = -\frac{\partial \phi_B}{\partial t}.}$$

$$\vec{\nabla}\times\vec{B} = \mu_0 J + \mu_0\epsilon_0 \frac{\partial \vec{E}}{\partial t}.$$

$$\int (\vec{\nabla}\times\vec{B})\, dS = \mu_0 \int \vec{J}\, d\vec{S} + \mu_0\epsilon_0 \frac{\partial}{\partial t}\int \vec{E}\cdot d\vec{S}$$

$$\boxed{\int \vec{B}\cdot d\vec{l} = \mu_0 I + \mu_0\epsilon_0 \frac{\partial \phi_E}{\partial t}}$$

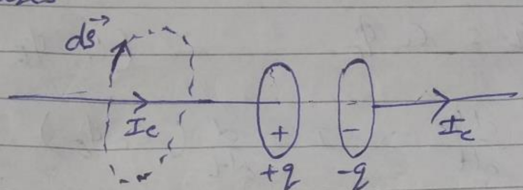
where $I = \int \vec{J}\cdot d\vec{S}$

$\phi_E = \int \vec{E}\cdot d\vec{S}$

Displacement Current :-

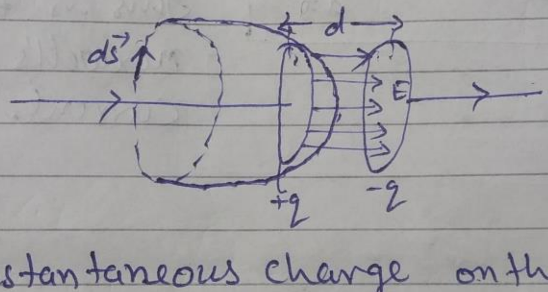
Displacement current is a quandity appearing in maxwell's equations that is define in terms of rate of change of electric displacement field.

Consider a charge capacitor. conducting wires bring current I_c on to one plate and away from the other as the charge on the plates increases



For above path we can apply Ampere's Law

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_c$$



The instantaneous charge on the capacitor plates is $q = C \Delta V$, where ΔV is the instantaneous potential difference across the plates.
As capacitor charge the electric field between the plates is changing.

$$C = \epsilon_0 \frac{A}{d}$$

where A is the area of the plate and d is the separation between the plates and C is the Capacitance of the Capacitor.

$$C = \epsilon_0 \frac{A}{d}$$

$$\Delta V = Ed$$

$$\therefore \quad q = C\,\Delta V$$

$$q = \epsilon_0 \frac{A}{\cancel{d}} \times E\cancel{d}$$

$$q = \epsilon_0 A E$$

$$q = \epsilon_0 \phi_E \qquad (\because \phi_E = EA)$$

Now

$$I_c = \frac{dq}{dt} \Rightarrow I = \frac{d}{dt}(\epsilon_0 \phi_E)$$

$$I_c = \epsilon_0 \frac{d\phi_E}{dt}$$

or

$$\boxed{I_D = \epsilon_0 \frac{d\phi_E}{dt}.}$$

This is known as Maxwell's displacement Current.