Dr. Mohammad Aslam, Dept. Of physics, B.Sc. part-2 physics (Hons) paper-iii, Lecture no-62

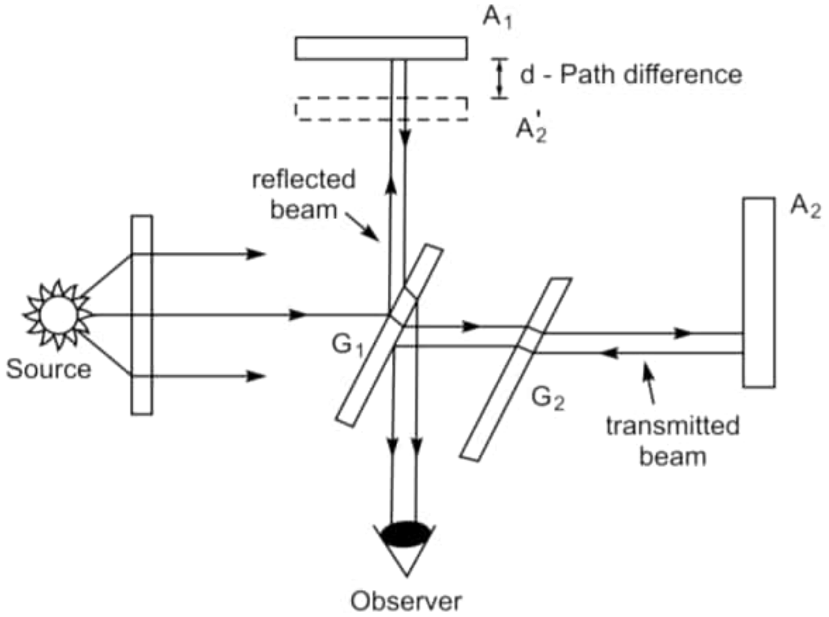
Michelson interferometer: theory

The Michelson interferometer employs a division of amplitude scheme. It can be used to carry out the following principal measurements:

- Width and fine structure of spectral lines.
- Lengths or displacements in terms of wavelengths of light.
- Refractive indices of transparent solids.
- Differences in the velocity of light along 2 different directions.

It operates as follows: we "divide" the wave amplitude by partial reflection using a beam splitter G1, with the two resulting wave fronts maintaining the original width by having reduced amplitudes [1]. A beam splitter is nothing more than a plate of glass, which is made partially reflective: as such, the splitting occurs because part of the light is reflected off of the surface, and part is transmitted through it.

The two beams obtained by amplitude division are sent in different directions against plane mirrors, then reflected back along their same respected paths to the beam splitter to form an interference pattern. The core optical setup, which is labelled in Fig.1, consists of two highly polished plates, A1 and A2, acting as the above-mentioned mirrors, and two parallel plates of glass G1 and G2 - one is the beam splitter, and the other is a compensating plate, whose purpose will be described below. The light reflected normally from mirror A1 passes through G1 and reaches the eye. The light reflected from the mirror A2 passes back through G2 for a second time, is reflected from the surface of G1 and into the eye.



The purpose of the compensating plate G2 is to render the path in glass of the two rays equal [1]. This is not essential for producing effective, sharp, and clear fringes in monochromatic light, but it is crucial for producing such fringes in white light (a reason will be given in the "White Light Fringes" section). The mirror A_1 is mounted on a carriage, whose position can be adjusted with a micrometer. To obtain fringes, the mirrors A_1 and A_2 are made exactly perpendicular to each other by means of the calibration screws (Fig. 1), controlling the tilt of A_2.

There are two very important requirements that need to be satisfied along with the above set up in order for interference fringes to appear:

1. Use an *extended light source*. The point here is purely one of illumination: if the source is a point, there is not much space for you to see the fringes on. You can convince yourself of the usefulness of using an extended source by positioning a variable size aperture in front of an extended source and shrinking its radius to the minimum possible (thus effectively converting it to a point source). As you can see, the field of view over which you can see the fringes shrinks right with it. Hence, it is in your best interest to use as big of a source as possible (a different screen is of further great aid here).

2. The light must be *monochromatic*, or nearly so. This is especially important if the distances of A_1 and A_2 from G_1 are appreciably different. The spacing of fringes for a given colour of light is linearly proportional to the wavelength of that light: hence the fringes will only coincide near the region where the path difference is zero. The solid line here corresponds to the intensity of interference pattern of green light, and the dashed curve — to that of red light. We can see that only around zero path difference will the colours remain relatively pure: as we move farther away from that region, colours will start to mix and become impure and unsaturated - already about 8-10 fringes away the colours mix back into white light, making fringes indistinguishable. Hence the region where fringes are visible is very narrow and hard to find with non-monochromatic light.

Some of the light sources suitable for the Michelson interferometer are a sodium flame or a mercury arc. If you use a small source bulb instead, a ground-glass diffusing screen in front of the source will do the job; looking at the mirror A_1 through the plate G_1, you then see the whole field of view filled with light.

Circular Fringes

To view circular fringes with monochromatic light, the mirrors must be almost perfectly perpendicular to each other. The origin of the circular fringes is understood from Fig. 2. The real mirror A_2 has been replaced by its virtual image A'_2 formed by the reflection in G_1: hence A'_2 is parallel to A_1.

Since light in the interferometer gets reflected many times, we can think of the extended source as being at L, where L is behind the observer as seen in Fig. 2; L forms 2 virtual images, L_1 and L_2, in mirrors A_1 and A_2', respectively. The virtual sources in L_1 and L_2 are said to be in phase with each other (such sources are called *coherent sources*), in that the phases of corresponding points in the two are exactly the same at all times. If d is the separation of A_1 and A_2', the virtual sources are then separated by $2d$, as can be seen in the diagram (Fig. 2).

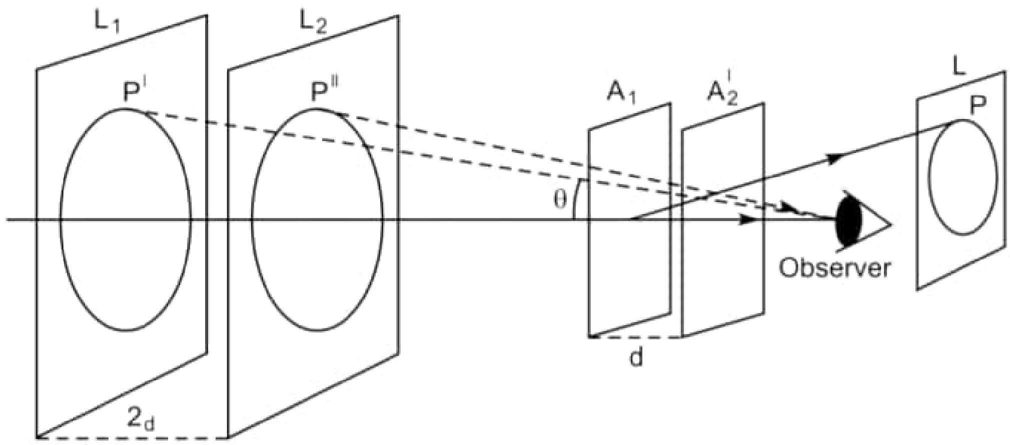


Figure 2: Virtual images from the two mirrors created by the light source and the beam splitter in the Michelson interferometer.

When d is exactly an integer number of half wavelengths, every ray that is reflected normal to the mirrors A_1 and A'_2 will always be in phase. The path difference, $2d$, must then be an integer number of wavelengths. Rays of light that are reflected at other angles will not, in general, be in phase. This means that the path difference between two incoming rays from points P' and P'' will be $2d\cos\theta$, where θ is the angle between the viewing axis and the incoming ray. We can say that θ is the same for the two rays when A_1 and A_2' are parallel, which implies that the rays themselves are parallel. Since the eye is focused to receive the parallel rays, it is more convenient to use a telescope lens, especially for looking at interference patterns with large values of d.

The parallel rays will interfere with each other, creating a fringe pattern of maxima and minima for which the following relation is satisfied:

$$2d\cos\theta = m\lambda \tag{1}$$

where d is the separation of A_1 and A'_2, m is the fringe order, λ is the wavelength of the source of light used, θ is as above (if the two are nearly collinear, we, of course, have $\theta \approx 0$ — this is the case for the fringes in the very centre of the field of view).

Since, for a given m, λ, and d the angle θ is constant, the maxima and minima lie on a circular plane about the foot of the perpendicular axis stretching from the eye to the mirrors. As was mentioned before, the Michelson interferometer uses division by amplitude scheme: hence the resultant amplitudes of the waves, a_1 and a_2, are fractions of the original amplitude A, with respective phases α_1 and α_2. We can calculate the phase difference between the two beams based on the respective mirror separation. If the path difference is $2d\cos\theta$, then the phase difference δ for light of wavelength λ is simply

$$\delta = 2\pi\frac{2d\cos\theta}{\lambda} \tag{2}$$

Here the ratio of the path difference to the wavelength tells you what fraction of a wavelength have you passed, and multiplication by 2π makes it a fraction of the full period of a sinusoid, thus giving you exactly the sought phase difference.

By starting with A_1 a few centimeters beyond A'_2, the fringe system will have the general appearance which is shown in Fig.3, where the rings of the system are very closely spaced. As the distance between A_1 and A'_2 decreases, the fringe pattern evolves, growing at first until the point of zero path difference is reached, and then shrinking again, as that point is passed.

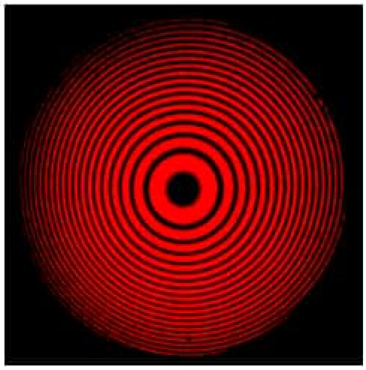


Figure 3: The circular fringe interference pattern produced by a Michelson interferometer.

This implies that a given ring, characterized by a given value of the fringe order m, must have a decreasing radius in order for (2) to remain true. The rings therefore shrink and vanish at the centre, where a ring will disappear each time $2d$ decreases by λ. This is because at the centre, $\cos\theta = 1$, and so we have the simplified version of equation (2),

$$2d = m\lambda \tag{3}$$

From here we see that the fringe order changes by 1 precisely when $2d$ changes by λ, hence for a fringe to disappear we need to decrease $2d$ by λ, as claimed above.