17/04/20

B.Sc. Part-2 Physics (Hons), Paper-III rd.

Lecture-9

1.

Wave Equations :-

The wave equation is an important second order linear partial differential equation for description of the wave. Such as mechanical wave or light wave,

ex

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{C^2}\frac{\partial^2 \psi}{\partial t^2}}$$ wave equation,

Electromagnetic wave in Dielectric media :-

Maxwell's Equation in charge free region

$\rho = 0, \quad J = 0$

$$\vec{\nabla}\cdot\vec{E} = 0 \quad \text{——— (I)}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{——— (II)}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{——— (III)}$$

$$\vec{\nabla}\times\vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{——— (IV)}$$

where μ and ϵ are the permiability and permitivitty of the medium,

Now take the curl of equation (III) we get

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$\nabla\cdot(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\frac{\partial}{\partial t}\left(\mu\epsilon\frac{\partial \vec{E}}{\partial t}\right)$$

$$0 - \nabla^2\vec{E} = \mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$+\nabla^2\vec{E} = +\mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\boxed{\nabla^2\vec{E} - \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0} \quad \text{--- (v)}$$

Similarly we can write wave equation in term of magnetic field $\vec{B}$.

$$\nabla^2\vec{B} = \mu\epsilon\frac{\partial^2\vec{B}}{\partial t^2}$$

$$\boxed{\nabla^2\vec{B} - \mu\epsilon\frac{\partial^2\vec{B}}{\partial t^2} = 0} \quad \text{--- (vi)}$$

The equations (v) and (vi) are necessary, but not sufficient constraints on the posible functions $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$.

The solution of the wave equations also satisfy maxwell's Equations.

By writing the vectors $\vec{E_0}$ and $\vec{K}$ (propagation vector) in term of components.

$$\vec{E_0} = (E_{0x}, E_{0y}, E_{0z})$$

$$\vec{K} = (K_x, K_y, K_z)$$

$$\vec{E}(\vec{r},t) = \vec{E_0}\cos(\omega t - \vec{K}\cdot\vec{r} + \phi_0)$$

$$\vec{\nabla}\cdot\vec{E} = 0$$

$$\vec{K}\cdot\vec{E_0}\sin(\omega t - \vec{K}\cdot\vec{r} + \phi_0) = 0$$

$$\vec{K}\cdot\vec{E_0} = 0$$

The field is perpendicular to the direction of motion it is a transverse plane wave.

Now $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$, $\vec{\nabla} \times \vec{B} = \mu \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

$$\vec{E} = \vec{E_0} \cos(\omega t - \vec{K} \cdot \vec{r} + \phi_0)$$
$$\vec{B} = \vec{B_0} \cos(\omega t - \vec{K} \cdot \vec{r} + \phi_0)$$

From the above Maxwell's Equations we can write:

$$\vec{\nabla} \times \vec{E_0} = -\frac{\partial \vec{B}}{\partial t}$$
$$\boxed{\vec{K} \times \vec{E_0} = \omega \vec{B_0}}$$

and $\vec{\nabla} \times \vec{B} = \mu \epsilon \frac{\partial \vec{E}}{\partial t}$

$$\boxed{\vec{K} \times \vec{B_0} = -\mu \epsilon \omega \vec{E_0}}$$

These constraints can be satisfied if the vector $\vec{K}$, $\vec{E_0}$ and $\vec{B_0}$ are mutually perpendicular.

The behaviour of a wave at a boundary between two media depends on the ratio of the electric field $\vec{E}$ and ~~to~~ magnetic field intensity $\vec{H}$.

The ratio of the amplitude of the electric field to the amplitude of the magnetic field intensity is called impedance "Z" of the medium.

using $B_0 = \mu_0 H_0$, $Z_0 = \frac{E_0}{H_0} = \sqrt{\frac{\mu_0}{\epsilon_0}}$

$$\boxed{Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}}$$

The impedance of free space is physically constant with value $Z_0 \simeq 376.7\,\Omega$.