B.Sc. Part-2 Physics (Hons) Paper-iii, Lecture-11 on the topic

“Energy density and momentum density of electromagnetic wave.”

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Lecture- 11

Energy density and momentum of electromagnetic wave:

we know that $U_E = \frac{1}{2}\epsilon_0 E^2$

and $U_H = \frac{1}{2}\mu_0 H^2$

But $\vec{B} = \mu_0 \vec{H}$

$$U_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

$$|B| = \frac{|E|}{c} = \sqrt{\mu_0\epsilon_0}\,|E|$$

electromagnetic energy.

$$U = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

$$U = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{\mu_0\epsilon_0}{\mu_0}E^2$$

$$U = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\epsilon_0 E^2$$

$$U = \left(\frac{1}{2}+\frac{1}{2}\right)\epsilon_0 E^2$$

$$\boxed{U = \epsilon_0 E^2}$$ electromagnetic energy density

Now $\vec{E} = \vec{E_0}\cos(kx - \omega t + \phi)$

$$E^2 = E_0^2\cos^2(kx - \omega t + \phi)$$

$$\boxed{U = \epsilon_0 E_0^2 \cos^2(kx - \omega t + \phi)}$$

we know the Poynting vector

$$\vec{S} = (\vec{E} \times \vec{H})$$

$$\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$$

$$\vec{S} = \frac{1}{\mu_0} E_0 \cos(kx - \omega t + \phi) B_0 \cos(kx - \omega t + \phi)$$

$$\vec{S} = \frac{1}{\mu_0} E_0 \sqrt{\epsilon_0 \mu_0} E_0^2 \cos^2(kx - \omega t + \phi) \hat{i}$$

$$\vec{S} = \frac{1}{\mu_0} C \frac{1}{C^2} E_0^2 \cos^2(kx - \omega t + \phi) \hat{i}$$

$$\vec{S} = \frac{C}{\mu_0} \mu_0 \epsilon_0 E_0^2 \cos^2(kx - \omega t + \phi) \hat{i}$$

$$\vec{S} = C \epsilon_0 E_0^2 \cos^2(kx - \omega t + \phi) \hat{i}$$

$$\vec{S} = C U \hat{i}$$

Now the momentum density is given as

$$\vec{P} = \frac{1}{C^2} \vec{S}$$

$$\vec{P} = \frac{1}{C^2} C U \hat{i}$$

$$\boxed{\vec{P} = \frac{U}{C} \hat{i}}$$

$\vec{P}$ is known as momentum density.