B.Sc. Part-1 Physics (Hons) Paper-ii, Lecture-15 on the topic

“Maxwell’s distribution in term of Energy and Velocity.”

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B.Sc- Part-1, Physics (Hons), Paper-IInd

Lecture-15

* Maxwell's distribution in term of energy and velocity

In the last lecture we have derived Maxwell's distribution law

$$n_i = g_i \, e^{-\alpha} \, e^{-\beta E_i}$$

$$\frac{n_i}{g_i} = e^{-\alpha} \, e^{-\beta E_i}$$

$$\boxed{f_{MB}(E) = e^{-\alpha} \, e^{-\beta E_i}} \qquad \left(\because \; f(E) = \frac{n_i}{g_i} \right)$$

now find the value of $e^{-\alpha}$,
the number of particles given by in term of Maxwell's distribution

$$n(E) = \rho(E) \, f_{MB}(E)$$

where $\rho(E)$ is the density of state

$$n(E) = \rho(E) \, f_{MB}(E)$$

$$N = \Sigma \, n(E) = \Sigma \, \rho(E) \, f_{MB}(E)$$

$$N = \int_0^\infty \rho(E) \, f_{MB}(E)$$

Now $\rho(E) = \dfrac{V}{2\pi^2 \hbar^3} \sqrt{2m^3} \sqrt{E} \, dE$

$$N = \frac{V}{2\pi^2 \hbar^3} \sqrt{2m^3} \int_0^\infty f_{MB}(E) \sqrt{E} \, dE$$

$$N = \frac{V}{2\pi^2 \hbar^3} \sqrt{2m^3} \int_0^\infty \sqrt{E} \, e^{-\alpha} \, e^{-\beta E} \, dE$$

$$N = \frac{V}{2\pi^2 \hbar^3} \sqrt{2m^3} \, e^{-\alpha} \int_0^\infty \sqrt{E} \, e^{-\beta E} \, dE$$

(2)

$$N = \frac{V}{2\pi^2\hbar^3}\sqrt{2m^3}\, e^{-\alpha}\int_0^\infty E^{1/2} e^{-\beta E}\, dE$$

Let $E^{1/2} = x$

$$\frac{1}{2}\frac{1}{E^{1/2}}\, dE = dx$$

$$dE = 2E^{1/2}\, dx$$

$$dE = 2x\, dx$$

$$N = \frac{V}{2\pi^2\hbar^3}\sqrt{2m^3}\, e^{-\alpha}\int_0^\infty x\, e^{-\beta x^2}\, 2x\, dx$$

$$N = \frac{V}{2\pi^2\hbar^3}\sqrt{2m^3}\, e^{-\alpha}\left\{2\int_0^\infty x^2 e^{-\beta x^2}\, dx\right\}$$

$$N = \frac{V}{2\pi^2\hbar^3}\sqrt{2m^3}\, e^{-\alpha}\left\{\sqrt{\frac{\pi}{4\beta^3}}\right\}$$

$$N = \frac{V}{2\pi^2\hbar^3}\sqrt{2m^3}\; e^{-\alpha}\sqrt{\frac{\pi}{4\beta^3}}$$

$$N = \frac{V}{4\pi}\sqrt{\frac{\cancel{4}16\pi^2\times 2m^3\times\pi}{\cancel{4}\beta^3}}\cdot\frac{1}{2\pi^2\hbar^3}\, e^{-\alpha}$$

$$N = \frac{V}{4\pi}\sqrt{\frac{8\pi^3 m^3}{\beta^3}}\,\frac{1}{2\pi^2\hbar^3}\, e^{-\alpha}$$

$$N = \frac{V}{8\pi^3}\left(\frac{2\pi m}{\beta\hbar^2}\right)^{3/2} e^{-\alpha}$$

$$e^{-\alpha} = \left(\frac{8\pi^3 N}{V}\right)\left(\frac{\beta\hbar^2}{2\pi m}\right)^{3/2}$$

but $\beta = \frac{1}{KT}$, $\frac{N}{V} = n$ number density

$$\boxed{e^{-\alpha} = \left((2\pi)^3 n\right)\left(\frac{\hbar^2}{2\pi m KT}\right)^{3/2}}$$

Now $f_{MB}(E) = e^{-\alpha} e^{-\beta E}$

$$\boxed{f_{MB}(E) = (2\pi)^3 n \left(\frac{h^2}{2\pi mKT}\right)^{3/2} e^{-\beta E}}$$

∴ Maxwell's distribution in term of energy

$E = \frac{1}{2} m v^2$

$$\boxed{f_{MB}(v) = (2\pi)^3 n \left(\frac{h^2}{2\pi mKT}\right)^{3/2} e^{-\frac{1}{2}\frac{mv^2}{KT}}}$$