B.Sc. Part-1 Physics (Hons) Paper-ii, Lecture-14 on the topic

“Maxwell’s distribution law.”

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Lecture-14 :->

* Maxwell's distribution law :->

Consider a system of 'N' particles. $\boxed{N}$ - system.

Let n_1 particles having energy E_1 and having degeneracy g_1,
n_2 particles having energy E_2 and having degeneracy g_2

⋮

n_K particles having energy E_K and having degeneracy g_K.

Now

Total number of particles, $N = \sum_{i=1}^{K} n_i$

then total energy of the system

$$U = \sum_{i=1}^{K} E_i n_i \quad \text{—— (1)}$$

A priori Probability of distribution (G),

$$G = g_1^{n_1} \cdot g_2^{n_2} \cdot g_3^{n_3} \cdots\cdots g_K^{n_K} \quad \text{—— (2)}$$

Now the thermodynamic probability (M),

$$M = \frac{N!}{n_1! . n_2! \cdots n_K!} \quad \text{—— (3)}$$

Total probability (ω),

$$\omega = M . G$$

$$\omega = \frac{N!}{n_1! . n_2! . n_3! \cdots n_K!} \cdot \left(g_1^{n_1} \cdot g_2^{n_2} \cdot g_3^{n_3} \cdots g_K^{n_K}\right)$$

$$\omega = \frac{N!}{\sum_{i=1}^{k} n_i!}\left(\sum_{i=1}^{k} g_i^{n_i}\right)$$

Taking log on both sides,

$$\ln\omega = \ln N! - \sum_{i=1}^{k} \ln n_i! + \sum_{i=1}^{k} n_i \ln g_i \quad \text{——} (4)$$

{Applying sterling's Approximation,

$$\ln x! = x\ln x - x$$ }

$$\ln\omega = N\ln N - N - \Sigma(n_i \ln n_i - n_i) + \Sigma n_i \ln g_i$$

$$\ln\omega = N\ln N - N - \Sigma n_i \ln n_i + \Sigma n_i + \Sigma n_i \ln g_i$$

but $N = \Sigma n_i$

$$\ln W = N\ln N - \cancel{N} - \Sigma n_i \ln n_i + \cancel{N} + \Sigma n_i \ln g_i$$

$$\ln W = \Sigma n_i \ln g_i + N\ln N - \Sigma n_i \ln n_i \quad \text{——} (5)$$

for most probable distribution,

$$d(\ln\omega) = 0$$

Now Equation 5 becomes

$$\delta \ln w = \Sigma \delta n_i \ln g_i + \Sigma n_i \delta \ln n_i - \Sigma \delta n_i \ln g_i$$

$$0 = \Sigma \delta n_i \ln g_i + \Sigma n_i \cdot \frac{1}{n_i} \delta n_i - \Sigma \delta n_i \ln g_i$$

$$0 = \Sigma \delta n_i \ln g_i - \Sigma \delta n_i \ln n_i + \delta n_i$$

$$0 = \Sigma \left(\ln \frac{g_i}{n_i}\right) \cdot \delta n_i \quad \text{— (6)}$$

From conservation of particles

$$\Sigma n_i = N \quad \text{and} \quad \Sigma \delta n_i = 0 \quad \text{— (7)}$$

From conservation of energy

$$\Sigma E_i n_i = U \quad \text{and} \quad \Sigma E_i \delta n_i = 0 \quad \text{— (8)}$$

multiply equation 7 by $(-\alpha)$ and equation 8 by $(-\beta)$ and adding to equation 6

$$\Sigma \left(\ln \frac{g_i}{n_i} - \alpha - \beta E_i\right) \cdot \delta n_i = 0$$

$$\therefore \ln \frac{g_i}{n_i} = \alpha + \beta E_i$$

$$\frac{g_i}{n_i} = e^{(\alpha + \beta E_i)}$$

$$\frac{n_i}{g_i} = e^{-(\alpha + \beta E_i)}$$

$$\boxed{n_i = g_i \cdot e^{-\alpha} \cdot e^{-\beta E_i}}$$

This equation called Maxwell's Boltzmann distribution law.