B.Sc. Part-2 Physics (Hons) Lecture-8 on the topic "Special Cases of Boundary Conditions."

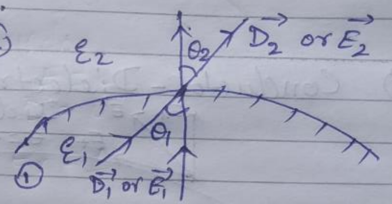
B.Sc. Part-2, Physics (Hons), Paper-III[rd]

Lecture-8

1

Special Case of boundary Condition :-

Boundary condition can be used to determine the refraction of the electric field across the interface.



According to boundary conditions

$$\left.\begin{aligned} E_{1t} &= E_{2t} \\ D_{1n} &= D_{2n} \end{aligned}\right\}$$

So

$$E_{1t} = E_{2t}$$

$$E_1 \sin\theta_1 = E_2 \sin\theta_2 \quad \text{--- (1)}$$

$$D_{1n} = D_{2n}$$

$$D_1 \cos\theta_1 = D_2 \cos\theta_2$$

$$\epsilon_1 E_1 \cos\theta_1 = \epsilon_2 E_2 \cos\theta_2 \quad \text{--- (2)}$$

divide equation (1) by equation (2).

$$\frac{E_1 \sin\theta_1}{\epsilon_1 E_1 \cos\theta_1} = \frac{E_2 \sin\theta_2}{\epsilon_2 E_2 \cos\theta_2}$$

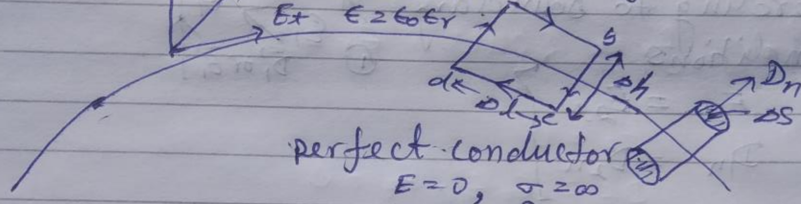
$$\frac{1}{\epsilon_1}\frac{\sin\theta_1}{\cos\theta_1} = \frac{1}{\epsilon_2}\frac{\sin\theta_2}{\cos\theta_2}$$

$$\frac{\tan\theta_1}{\epsilon_1} = \frac{\tan\theta_2}{\epsilon_2}$$

$$\frac{\tan\theta_1}{\epsilon_0 \epsilon_{r_1}} = \frac{\tan\theta_2}{\epsilon_0 \epsilon_{r_2}}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r_1}}{\epsilon_{r_2}}}$$ — Law of refraction of the electric field at the boundary free of charge $\rho_f = 0$.

② Conductor - Dielectric Boundary Conditions :→



apply $\oint \vec{E}\cdot\vec{dl} = 0$

$$\int_a^b \vec{E}\cdot\vec{dl} + \int_b^c \vec{E}\cdot\vec{dl} + \int_c^d \vec{E}\cdot\vec{dl} + \int_d^a \vec{E}\cdot\vec{dl} = 0$$

$$E_t \Delta l + E_n \frac{\Delta h}{2} + 0 - 0 + 0 + E_n \frac{\Delta h}{2} = 0$$

$$E_t \Delta l = 0$$

$$\boxed{E_t = 0} \text{ —— (i)}$$

apply $\oint \vec{D}\cdot\vec{dS} = Q_{enc}$

$$\int_{top} \vec{D}\cdot\vec{dS} + \int_{bottom} \vec{D}\cdot\vec{dS} = Q_{enc}$$

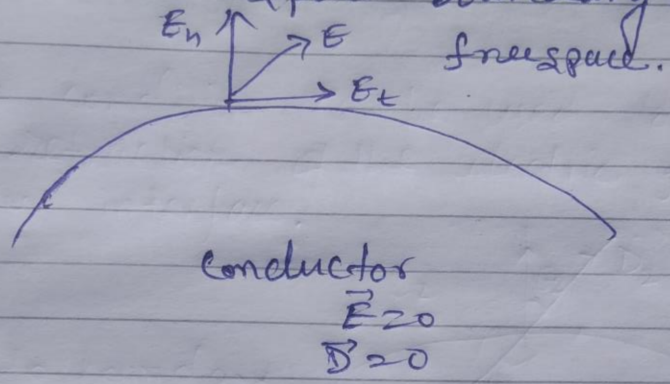
$$D_n \Delta S - 0 = \rho_s \Delta S \qquad (\because Q = \rho \Delta S)$$

$$\boxed{D_n = \rho_s} \text{ —— (ii)}$$

$$D_n = \rho_s$$

$$\boxed{\epsilon_0 \epsilon_r E_n = \rho_s}$$

③ Conductor - Free space boundary condition:



For free space $\epsilon_r = 1$

$E_t = 0$

$D_t = \epsilon_0 \epsilon_r E_t = 0$

$D_t = \epsilon_0 E_t = 0$

$D_t = 0$

$D_n = \epsilon_0 E_n = \rho_s$

$D_n = \rho_s$

The electric field $\vec{E}$ must be external to the conductor and normal to it's surface.